# 2-day meeting in Stuttgart on Computational Lie Theory

5-6 July 2018

## SCHEDULE

### THURSDAY 5th OF JULY

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|---|---|
| 10:00–10:35 | **Willem De Graaf** |
| 10:35–11:05 | Coffee break |
| 11:05–11:40 | **Goetz Pfeiffer** |
| 11:45–12:20 | **Ivan Marin** |
| 12:20–14:00 | Lunch |
| 14:00–14:35 | **Gunter Malle** |
| 14:40–15:15 | **Maria Chlouveraki** |
| 15:15–15:45 | Coffee break |
| 15:45–17:30 | **Discussion Session** |

### FRIDAY 6th OF JULY

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| 10:00–10:35 | **SungSoon Kim** |
| 10:40–11:15 | **Frank Lübeck** |
| 11:15–11:45 | Coffee break |
| 11:45–12:20 | **Patrick Dehornoy** |

## TITLES & ABSTRACTS

### Maria Chlouveraki: The symmetrising trace conjecture for Hecke algebras

Exactly twenty years ago, Broué, Malle and Rouquier published a paper in which they associated to every complex reflection group two objects which were classically associated to real reflection groups: a braid group and a Hecke algebra. Their work was further motivated by the theory, developed together with Michel, of "Spetses", which are objects that generalise finite reductive groups in the sense that their associated Weyl groups are complex reflection groups. The four of them advocated that several nice properties of braid groups and Hecke algebras generalise from the real to the complex case, culminating in two main conjectures as far as the Hecke algebras are concerned: the "freeness conjecture" [BMR] and the "symmetrising trace conjecture" [BMM]. The two conjectures are the cornerstones in the study of several subjects that have flourished in the past twenty years, but had remained open until recently for the exceptional complex reflection groups. In the past five years, the proof of the "freeness conjecture" was completed for all exceptional complex reflection groups. In this talk, we will discuss our proof of the "symmetrising trace conjecture" for the first five exceptional groups. This is joint work with Christina Boura, Eirini Chavli and Konstantinos Karvounis.

**Willem De Graaf: Nilpotent orbits of real and complex symmetric pairs**

Symmetric pairs are constructed from $\mathbb{Z}/2\mathbb{Z}$-gradings of semisimple Lie algebras. They yield many examples of representations of reductive algebraic groups. An orbit of such a group is called nilpotent if its closure contains 0. We consider this construction over the fields $\mathbb{C}$ and $\mathbb{R}$ and look at the problem of listing the nilpotent orbits. It turns out that this is considerably more difficult over $\mathbb{R}$ than over $\mathbb{C}$. The classification of the nilpotent orbits will be given for a symmetric pair obtained from the simple Lie algebra of type $D_4$. This yields a representation of the group $SL(2,F)^4$ on a 16-dimensional space ($F = \mathbb{R}, \mathbb{C}$) which is of interest for the study of black holes. (This is joint work with Heiko Dietrich, Daniele Ruggeri, and Mario Trigiante.)

**Patrick Dehornoy: A conjecture about enveloping groups of gcd-monoids**

We conjecture that Derek Holt's padded version of multifraction reduction solves the word problem of the enveloping group of every gcd-monoid, resulting in an effective solution if, and only if, the needed amount of padding is bounded above by a Turing computable function. In the case of Artin-Tits groups, it is known so far that a zero padding is enough in the FC case, and that a quadratic padding is enough in the sufficiently large type.

**SungSoon Kim: Fully commutative elements of the Coxeter group of type B and beyond**

Fan-Graham-Stembridge classified the Coxeter groups having the finite number of fully commutative elements (Fan-Graham for simply-laced cases, Stembridge for all types of Coxeter groups, '96 '98). Feinberg-Lee ('12,'14) gave a new combinatorial realization of fully commutative elements for types $A_n$ and $D_n$. In this talk, we recall the results of Feinberg-Lee's work to generalize to the Coxeter group of type $B_n$ and beyond to the complex reflection group of type $G(d, 1, n)$. In this talk, we recall the results of Feinberg and Lee and study analogous results for the complex reflection groups of types $G(d, 1, n)$, $G(d, d, n)$ and $G(d, r, n)$. In particular, when $d = 2$, we characterize the fully commutative elements of the Coxeter group of type $B_n$ and recover Feinberg and Lee's results for type $D_n$ (joint work with G. Feinberg, K.H.Lee, S.-J. Oh '16, '18).

**Frank Lübeck: Turning weight multiplicities into Brauer character tables**

In this talk we consider the large groups of Lie type whose ordinary character tables are in the ATLAS. I will sketch two methods to compute their Brauer character tables with respect to their defining characteristic.

The ingredients are information about the irreducible rational representations of the corresponding algebraic groups in form of weight multiplicities, parameterizations of semisimple conjugacy classes, and ad hoc techniques to find fusions of conjugacy classes from the Brauer table to the known ordinary character table.

### Gunter Malle: On the number of characters in a block

Brauer's long-standing $k(B)$-conjecture gives a bound on the number of ordinary characters in a $p$-block of a finite group. We report on recent progress on this conjecture for blocks of quasi-simple groups and also discuss two related estimates involving further block theoretic invariants that we had proposed jointly with Navarro.

### Ivan Marin: Towards a Krammer representation for complex braid groups

I will expose the general program of constructing a generalization of the (faithful) linear representation of the usual braid group investigated by Krammer to other complex reflection groups, and present the recent results of Z. Chen and of my student G. Neaime in this direction.

### Goetz Pfeiffer: Bisets and the Double Burnside Algebra of a Finite Group

The double Burnside group $B(G, H)$ of two finite groups $G, H$ is the Grothendieck group of the category of finite $(G, H)$-bisets. Certain bisets encode relationships between the representation theories of $G$ and $H$. Bouc's biset category provides a framework for studying such relationships, it has finite groups as objects, and $B(G, H)$ as morphisms between $G$ and $H$, with composition induced by the tensor product of bisets. The endomorphism ring $B(G, G)$ is called the double Burnside ring of $G$. In contrast to the (ordinary) Burnside ring $B(G)$, the double Burnside ring $B(G, G)$ of a nontrivial group $G$ is not commutative. In general, little more is known about the structure of $B(G, G)$.

In the talk I'll describe a relatively small faithful matrix representation of the rational double Burnside algebra $\mathbb{Q}B(G, G)$ for certain finite groups $G$, based on a recent decomposition of the table of marks of the direct product $G \times G$, exhibiting the cellular structure of the algebra $\mathbb{Q}B(G, G)$. This is joint work with Sejong Park.